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Modelling rolling contact fatigue cracks in the hydrodynamic lubrication regime: a coupled approach

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Abstract

In this paper we present a coupled method for modelling fluid-solid interactions within a crack generated by rolling contact fatigue (RCF) in the presence of a lubricant. The technique describes the fluid flow in the contact area and within a surface-breaking crack and explores how this affects the elastic deformation of the solid while the moving load traverses the surface crack. The approach sheds light on the transient response of the system by explicitly modelling the localised fluid flow and support within the crack, providing a physically-accurate description of the phenomenon under investigation.

Keywords: *Rolling Contact Fatigue (RCF), Elastohydromdynamic lubrication, Boudary Element Modelling (BEM)*

1. Introduction

There has been much speculation as to the precise role that the liquid plays in the failure process in rolling contacts, and given its importance for how the nature and properties of any lubricant might affect the fatigue life. A number of experimental and theoretical studies [1-6] conducted over the last three decades have suggested a number of hypotheses explaining how the liquid can influence fatigue crack growth: (i) by reducing friction between crack faces (“friction reduction” mechanism); (ii) by applying direct pressure on the crack faces as fluid flows into the crack (“hydraulic pressure” mechanism); (iii) by the “fluid entrapment effect” which causes hydrostatic pressure at the crack tip. These three mechanisms are quasi-static in nature. A fourth mechanism based on “the squeeze fluid layer” [7] includes transient effects taking place within the crack.

To study crack propagation in lubricated rolling contacts it is necessary to couple a fluid model, in this case a hydrodynamic model, with the response of the cracked solid by invoking the principles which govern linear elastic fracture mechanics. Among the existing models, both the “fluid entrapment” and the “squeeze fluid layer” theories are based on a grounded physical understanding of the phenomenon under investigation. However, no attempt has been successful in characterising the coupled behaviour of the cracks formed and propagating in lubricated contacts. A major shortcoming of the existing approaches is the lack of coupling between the fluid and solid solvers implemented to model lubricated contacts in the presence of cracks. In this paper, we develop the coupled fluid-solid algorithm proposed by the authors [8] to study a hydrodynamic contact passing over a surface-breaking crack.

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2. Formulation

The physical problem considered in this paper is shown in Fig. 1(a). A cracked semi-infinite body is in contact with a loaded (W) cylindrical roller in the presence of an iso-viscous, Newtonian lubricant where the crack is inclined at 25° from the horizontal plane. The cylindrical roller is modelled, in first approximation, using a flat convergent surface (see Fig. 1(b)). The divergent section of the roller is ignored because the fluid experiences cavitation in this region which is assumed to generate a zero pressure condition and therefore does not contribute to the solid deflection. In addition the crack enters the contact area in a fully flooded state and once inside the contact area the flow between the crack and the surface is defined by the pressure gradient experienced at the crack mouth.

Two independent solvers, a fluid solver based on a finite volume (FV) representation of the Reynolds' equation [9] and an elastic solid solver based on the distributed dislocation technique [10], are coupled at the liquid/solid interface. The transient interaction between the algorithms is achieved using an iterative scheme where the converged solution is obtained both in terms of pressures (p) and the deflections within the crack (U_n).

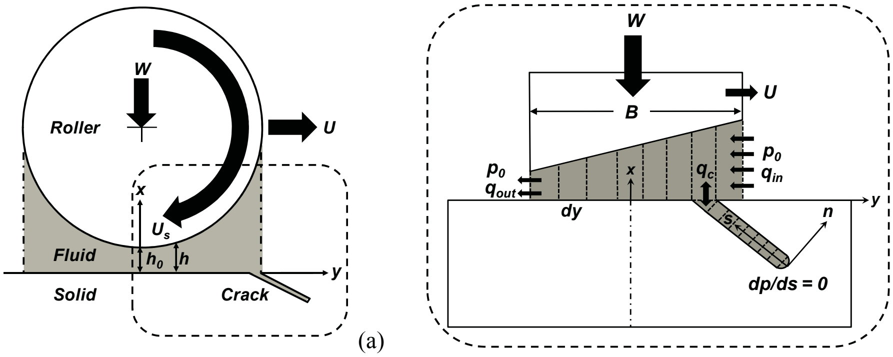

Figure 1 – Schematics: (a) the full system under investigation and (b) the fluid domain solved using finite volumes.

In developing the algorithm the authors have first considered the model proposed in [7] for solving fluid pressure in a fully flooded crack problem using a squeeze film model. This was then extended to include a bearing moving above a lubricated crack by adopting a FV solver [8]. The main limitation of using a squeeze film model and only considering the lubricant inside the crack (in isolation) is the assumption that one has to make about the pressure at the crack mouth, p_m (*e.g.* see Bogdánski [7]). The use of this condition means that no restriction is placed on the volumetric flux, q_c, exchanged between the surface fluid film and the deforming crack. This renders the problem inherently uncoupled and can lead to the calculation of unrealistic pressure profiles within the crack – to satisfy the pressure boundary condition an unregulated flux would tend to "inflate" the crack until an equalized pressure (equal to the boundary pressure p_m) is reached at convergence.

The methodology proposed by the authors overcomes this problem by limiting the flow at the crack mouth by coupling not only the pressure but also the volumetric flux between the lubricant film formed outside and within the crack. The crack and surface film regions are discretised into a series of finite volumes (see Fig. 1(b)), similar to Arghir [9], where the volumetric flux, q, is calculated at each element boundary. A coupling term relates the fluid flux in/out of the crack, q_c, to the pressure gradient, dp_m /ds, and crack film thickness at the mouth. Therefore, the only boundary conditions required are those at the edges of the contact and at the crack tip (with no need to specify either the pressure or its gradient anywhere within the crack!). These are respectively atmospheric pressure, p_0, outside the contact and a zero pressure gradient ($dp / ds=0$) at the crack tip (see Fig. 1(b)).

Turning now to the solid and fluid solvers and the coupling strategy, the fluid domain is expressed as a set of linear equations connecting the pressures at the centre of each volume which can be solved using a robust iterative method [11]. The contact bodies are considered to be two elastic half-planes in plane strain and the solution of the crack problem is achieved using the distributed dislocation technique (DDT) [10]. The problem is considered to be linear elastic in nature because, although localised plasticity can occur in the region of the crack tip, geometry and loading conditions of interest are such that small scale yielding applies. Therefore, the formulation of the solid solver can be broken down into three subsets, the individual solutions of which are combined using superposition.

Initially the stress and displacement fields in the neighbourhood of the crack due to the surface load and the internal crack pressure are found in the absence of the crack. The boundary condition at the crack faces, namely zero shear tractions and normal tractions being equal to the applied fluid pressure, are then satisfied deploying distributed dislocations (or strain nuclei) of unknown densities along the crack [10]. The stress and displacement fields within the half-plane for the un-cracked configuration are obtained using two semi-analytical formulations:

a. Muskelishvili's potential theorem is used to find the stresses and displacements induced by a piecewise triangular linear discretisation of the pressure profile acting on the half-plane as a result of the moving contact load [12];
b. Melan's solution for a point load acting at a generic location within the half-plane is employed as a Green's function to solve for the stress and displacement field generated by the internal crack pressure [13].

Once the unknown dislocation densities which satisfy the traction boundary conditions at the crack faces and their related stress and displacement fields are known, the full solution of the elastic problem can be found through the superposition of the three stress and displacement subsets. Mode I and Mode II stress intensity factors can also be obtained directly via Krenk's interpolation [10].

Coupling between the fluid and the solid solvers is subsequently achieved by modifying the film thicknesses in the fluid solver, $h(y,s)$, using the deformation at the crack surfaces (opening/closing) and the deflection at the contact surface. The iteration between the two solvers is continued until compatibility between the fluid pressures and the film thickness is achieved everywhere. The next time-step is then considered ($t \rightarrow t+1$) and the position of the crack relative to the contact centre updated. This process is repeated until the crack has traversed the loaded area. At each instant, t, the converged solution at the previous time step, $t-1$, is used to initialise the solvers.

3. Validation and Discussion

It is important to consider the approximations made to model the problem under consideration, and, in particular, the boundary conditions used to solve the fluid problem: (i) the geometry; (ii) fluid film boundaries.

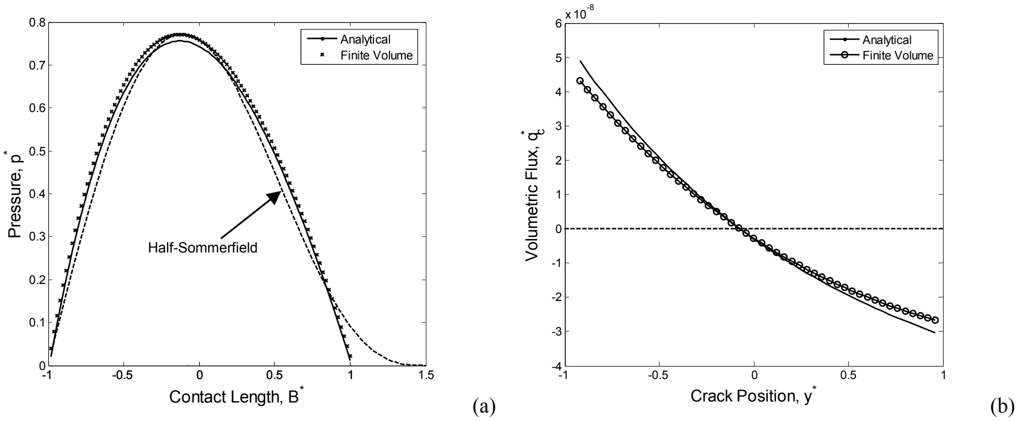

Figure 2. (a) Surface pressure: comparison between FV, analytical convergent wedge and half-Sommerfeld solutions and (b) Volumetric flux at the crack mouth: FV vs. Analytical solution. ***The subscript * refers to normalized quantities.***

<u>*The bearing geometry*</u>: The impact of the approximating the curvature of the contacting cylinder by a linear convergent wedge on the solution of the problem has been tested. In Fig. 2(a) the pressure distribution produced by a linear wedge in the absence of the crack is compared to that produced by the equivalent "half-Sommerfeld" [14] solution for a cylinder on a flat. The overall agreement between the two solutions supports the use of this approximation for development purposes. In addition, the comparison between the numerical (FV) and the analytical solutions for a linear convergent bearing is shown. The good agreement between the solutions supports the use of the FV technique implemented by the authors.

<u>*Inlets, outlets and coupling*</u>: Each of the two fluid films (surface; crack) requires the application of boundary conditions at each inlet and outlet. For the surface film this is achieved by applying a constant pressure at each end of the linear convergent bearing. These conditions correspond to assuming that the contact is fully flooded such that sufficient lubricant is present at the inlet for the supply not to be cut off and that this lubricant is subject only to atmospheric pressure. In the crack a zero pressure gradient is applied at the crack tip, $s=0$. This corresponds to the fact that at the crack tip no flow may occur due to the presence of the solid boundaries and consequently the pressure gradient must be zero. This leaves only the conditions used to couple the fluid films to be examined. The

coupling between the films under the bearing and within the crack is achieved through the flux term quantifying the flow of lubricant from the crack to the surface film, q_c,. It is therefore important to consider the validity of using such flux-based coupling. A comparison between the change in volume of the fluid within the crack (Analytical) and the flux at the crack mouth computed at the merging element between the fluid film under the bearing and the fluid within the crack (Finite Volume) is shown in Fig. 2(b). It should be noted that, although the trends and magnitudes of the two measures are similar, the two curves do not show perfect agreement.

In order to shed light on the nature of such apparent mismatch, the solid model needs to be considered. Of critical importance is the ability of the model to predict correctly the deformation of the crack faces. Due to a cubic relationship between the deflection and the flow, incorrect prediction of the crack mouth deflection will impact heavily on the accuracy of the flux calculation. A comparison between the semi-analytical (DDT) model having 60 (used for the calculations performed here) and 90 integration points and a refined plane strain FE model (ANSYS 11.0) is shown in Fig. 3(a) for a constant pressure applied within the crack faces. The discrepancy in terms of normalized deflections at the crack mouth due to lack of resolution and to the interpolation of the end point behaviour within the DDT method may play an important role in the computation of the coupling flux term. This suggests that the implementation of a finer grid may be required in the solid solver to achieve improved accuracy.

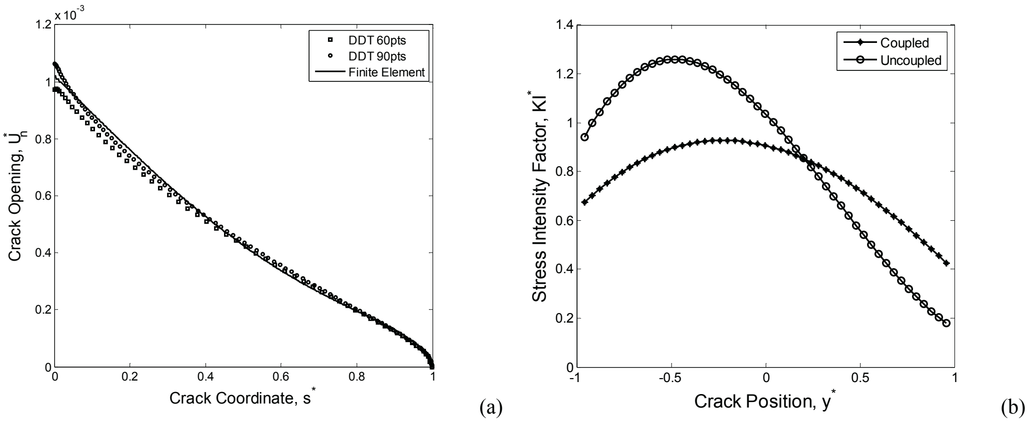

Figure 3. (a) Crack deflection: DDT vs. FEM and (b) Mode I stress intensity factor: Coupled vs. Uncoupled models.

Finally, Fig. 3 (b) shows the comparison between the normalized mode I stress intensity factor, K^*_I, for the problem under consideration using both the present fully coupled approach and an uncoupled solution similar to that proposed in [7]. It is clearly shown that the coupled model produces both a reduced maximum and a reduced range of K^*_I values. These results are particularly relevant if crack propagation shows Paris law-type behavior, and the stress intensity factor range governs crack propagation.

Acknowledgements

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